Problem 1.6

Add two complex vectors $(2+3i)$ and $(4-i)$, expressing the result as $A\angle\theta$.

Solution

Let $z_1 = 2+3i$ and $z_2 = 4-i$. Then

$$\begin{aligned} z_1 + z_2 &= (2+3i) + (4-i) \\ &= (2+4) + (3i-i) \\ &= 6+2i \end{aligned}$$

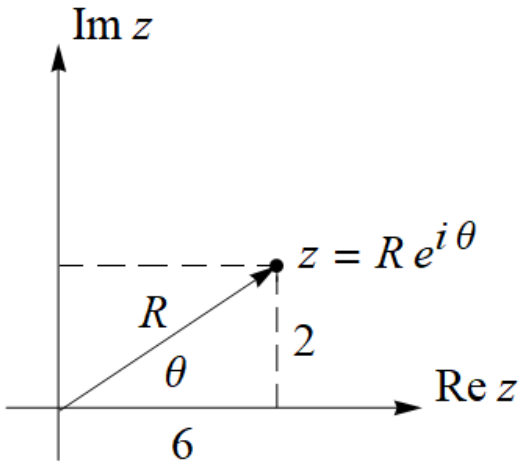

Figure 1: This figure shows the vector $z = 6+2i$ in the complex plane.

$$\begin{aligned} R &= \sqrt{6^2+2^2} = \sqrt{40} = 2\sqrt{10} \\ \theta &= \tan^{-1}\frac{2}{6} = \tan^{-1}\frac{1}{3} \approx 0.3218 \end{aligned}$$

The representation in polar form is then

$$6+2i = 2\sqrt{10}\exp\left(i\tan^{-1}\frac{1}{3}\right).$$

Therefore,

$$6+2i = 2\sqrt{10}\angle\ \tan^{-1}\frac{1}{3}.$$